INTERNAL ASSESSMENT

How does the change in mass (m) attached to a mass-spring system affect its period (T) of oscillation?

IB Physics

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Part I: Introduction

1.1 Aim: Determining the elastic constant (k) of the spring of a mass-spring system.

1.2 Research Question: How does the change in mass (m) attached to a mass-spring system affect its period (T) of oscillation?

1.3 Overview: My investigation focuses on validating a given elastic constant of a 5 Newtons dynamometer, varying the mass of the object attached to the spring and the time of the oscillations. When I first began having IB HL Physics Classes I got interested in the topic of Simple Harmonic Motion. One day, in one of our classes about this topic, we were studying the mass-spring equation, and my teacher said that elastic constants vary depending on the springs/dynamometer, but we didn't do any further experimentation using this equation. With that, I got frustrated because I got interested in the subject and wanted to make an experiment to better understand/visualize what we were studying. Talking to my teacher, he said that this could be a good exploration for me to do on my own.

1.4 Background Information: A mass-spring system, in a few words, is a system of masses attached to springs that oscillate in a simple harmonic motion. Simple harmonic motion is defined as the "motion in which the acceleration (a) is proportional to the displacement (x) from a fixed point and is always directed towards that fixed point"[1], algebraically it is:

$$a = -\omega^2 x \tag{1}$$

[1] This paper used Homer, D. & Bowen-Jones, M. *Physics* (2014) book for its completion.

Within simple harmonic motion, one of the most important concepts is the angular frequency (ω). By definition, "it is the rate of change of the angle with time", it is important to notice that systems that happen through simple harmonic motion move in oscillations, and that normally these oscillations have their displacements associated with angular positions, for example, half an oscillation would be an angular displacement of π radians and, for a full oscillation, it would be 2π radians. The equation of the angular frequency is:

$$\omega = \frac{2\pi}{T} \tag{2}$$

Each oscillation has a certain time to be fulfilled and this is called the period (T). Returning to the mass-spring system, each spring has a specific elastic constant – which is the "measure of the spring's stiffness" (UNIVERSITY OF TENNESSEE) and is extended by "x" from its equilibrium position. When the spring is being extended, a restoring force is going to act on the mass (m) in the opposite direction to the displacement, as shown by Hooke's Law:

$$F = -kx \tag{3}$$

As the elastic force is acting as a net force on the object, it's possible to substitute *F* according to Newton's second law:

$$ma = -kx \tag{4}$$

Rearranging:

$$a = -\left(\frac{k}{m}\right)x \tag{5}$$

The equations (1) and (5) can be compared to finding the following correspondence:

$$\omega^2 = \left(\frac{k}{m}\right) \tag{6}$$

Knowing that the period for SHM is given by equation 2, rearranging it:

$$T = \frac{2\pi}{\omega} \tag{7}$$

Substituting and rearranging, the equation of the period of the mass-spring can be found:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{8}$$

The greater the mass, the greater the period, but the greater elastic constant, the smaller the period. The equation for the mass-spring system contains all three of the previously stated variables. To discover what the elastic constant of this dynamometer is, firstly it is needed to linearize this equation, this means, write it in the format of:

$$y = ax + b \tag{9}$$

Rearranging and linearizing the equation (8), we get:

$$T^2 = \frac{4\pi^2}{k}m + 0 \tag{10}$$

(which means that the square of the period is proportional to the mass)

Thus, the slope is going to be:

$$\text{Slope} = \frac{4\pi^2}{k} \tag{11}$$

With that, equaling the value of the slope found to $\frac{4\pi^2}{k}$, the value of the elastic constant can be given.

Part II: Investigation

2.1 Materials:

- Stopwatch
- Dynamometer
- Water bottle
- Water to fill up the bottle
- Weighing scale
- Desk to attach the dynamometer
- Ruler

Figure 1: Picture of the structure used by the candidate

2.2 Method:

- **Independent variable:** Mass (m)
    - 0.100 kg, 0.147 kg, 0.197 kg, 0.251 kg, 0.297 kg, 0.347 kg, 0.402 kg, 0.437 kg, 0.463 kg, 0.496 kg.
- **Dependent variable:** Period of the oscillation (T)
- **Controlled variable:** Temperature (°C)
    - The temperature in the room must remain the same because the stiffness of the spring depends on it. If the room gets colder it will get stiffer. (WERNER et al., 2015)

2.3 Procedure: This experiment (Figure 1) consisted of a water bottle attached to a dynamometer which was fixed on a desk. When the bottle is displaced from its equilibrium position, it starts performing a simple harmonic motion.

To make the experiment:

1. Build the experiment structure as represented in Figure 1;
2. Fill up a water bottle with one predetermined mass;
3. Measure the mass of the bottle with water using a weighing scale with 0.001 kg precision;
4. Attach the top of the water bottle to the dynamometer;
5. Hold the water bottle with one hand, and hold the stopwatch with the other hand;
6. Release the water bottle, measure the time of 10 oscillations, and repeat this process 3 times;
7. Repeat these steps for the remaining 9 masses.

2.4 Risk Assessments: Since this experiment was done by attaching the dynamometer to a desk, there were no risks of someone getting hit by the mass attached to the dynamometer. If it was done with higher masses it would be needed to attach the dynamometer to a higher place, with that, it would be necessary to pay attention to not hit someone.

2.5 Data Analysis:

The elastic constant of the dynamometer – which is the value this experiment aims to get as close as possible, can be previously calculated. As the dynamometer has 5.00 ± 0.03 N, and is 12.00 ± 0.05 centimeters long – the uncertainties were determined as half of the smallest division on an analogue scale –, and knowing that elastic constant is found by rearranging equation 3, it is just a matter of substituting the known values:

Equation 3 rearranged:

$$k = \frac{F}{x}$$

Substituting the values, using SI units:

$$k = \frac{5}{0.12}$$

$$k = 41.7\ Nm^{-1}$$

To find its uncertainty, the sum of the frictional uncertainties of the force – F – and the displacement – x – must be multiplied by the value of the elastic constant – k:

$$\Delta k = \left[\frac{\Delta F}{F} + \frac{\Delta x}{x}\right] \cdot k$$

Substituting the values:

$$\Delta k = \left[\frac{0.03}{5} + \frac{0.05}{12}\right] \cdot 41.7$$

$$\Delta k = 0.01 \cdot 41.7$$

$$\Delta k = 0.4\ Nm^{-1}$$

To get as close as possible to the reference value, ten different masses were used in the experiment to find ten different periods (Table 1), with those values the value of k could be found graphically using equation 10.

It is also important to notice that for the whole experiment the uncertainty of time was considered the reaction time, which is $\pm$ 0.250 seconds (Human Benchmark, 2020). This can be considered because the only situation where uncertainty can be appointed is when the stop-button on the Stopwatch needs to be pressed.

Sample of how to do the calculations needed:

Processed data:

Average time of 10 oscillations = $\frac{(t_1+t_2+t_3)}{3} \rightarrow \frac{(3.2+3.1+3.1)}{3}$ = *3.1 s*

Period of one oscillation = $\frac{(Average\ time\ of\ 10\ oscillations\ )}{10} \rightarrow \frac{(3.1)}{10}$ = *0.31 s*

Period squared = $(Period\ of\ one\ oscillation)^2 \rightarrow (0.31)^2$ = *0.10 s^2*

Uncertainty of the processed data:

Period of one oscillation = $\frac{(reaction\ time)}{10} \rightarrow \frac{(0.3)}{10}$ = *0.03 s*

Period squared = $T^2 \cdot 2 \cdot \Delta T = 0.48^2 \cdot 2 \cdot 0.03$ = *0.01 s^2*

Obs.: To have only one uncertainty for the period squared value, the highest possible value for the period was used in the calculation.

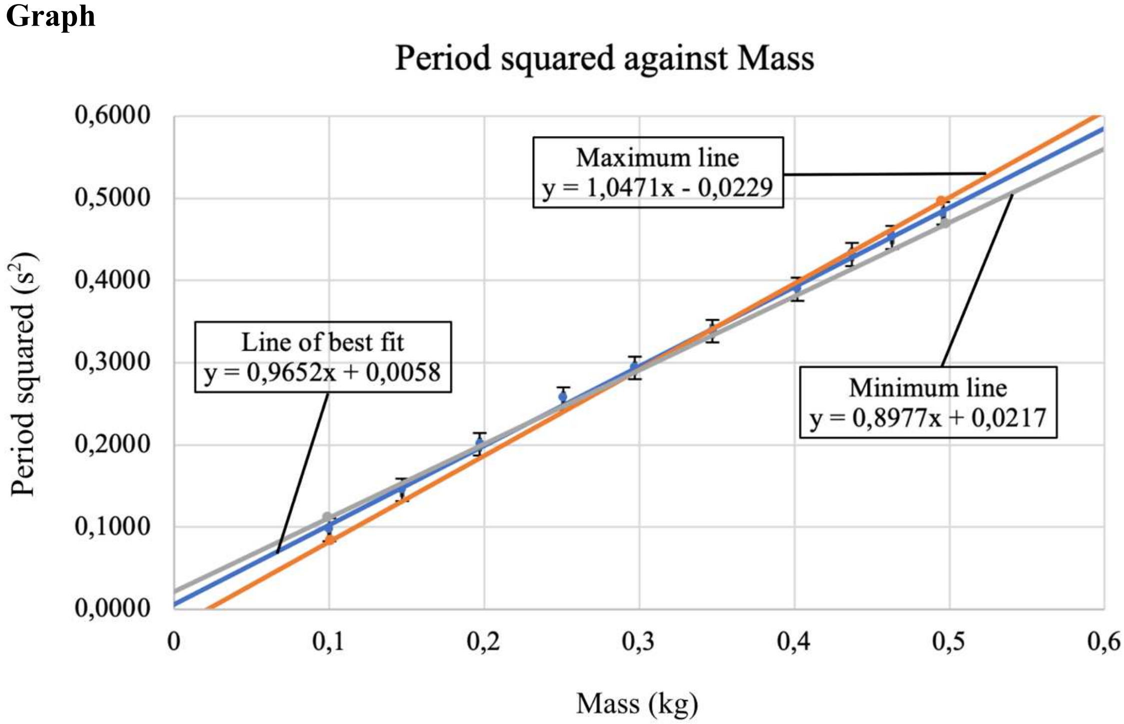
Table 1: Table showing the raw data collected

RAW DATA			
Mass m $\pm$ 0.001 kg	Time 1 $t_1 \pm 0.3$ s	Time 2 $t_2 \pm 0.3$ s	Time 3 $t_3 \pm 0.3$ s
0.100	3.2	3.1	3.1
0.147	3.8	3.8	3.9
0.197	4.6	4.5	4.4
0.251	5.1	5.0	5.1
0.297	5.3	5.4	5.5
0.347	5.8	5.9	5.8
0.402	6.3	6.2	6.2
0.437	6.5	6.6	6.6
0.463	6.8	6.7	6.7
0.496	7.0	6.9	6.9

Table 2: Table showing the processed data

PROCESSED DATA		
Average time of 10 oscillations $t \pm 0.3$ s	Period of one oscillation $T \pm 0.03$ s	Period squared $T^2 \pm 0.01$ s^2
3.1	0.31	0.10
3.8	0.38	0.15
4.5	0.45	0.20
5.1	0.51	0.26
5.4	0.54	0.29
5.8	0.58	0.34
6.2	0.62	0.39
6.6	0.66	0.43
6.7	0.67	0.45
6.9	0.69	0.48

2.6 Graph



Graph 1: Graph representing the values of the period squared against the mass

2.7 Graph Analysis: The line of best fit is the trendline of the points without uncertainties and has a slope that values 0.9652 s^2/kg. The Maximum line is the trendline that has the points (0.101 kg; 0.0829 s^2) and (0.495 kg; 0.4955 s^2), and the slope of 1.0471 s^2/kg. Lastly, the Minimum line is the trendline that has the points (0.99 kg; 0.1105 s^2) and (0.497 kg; 0.4678 s^2), which has the slope of 0.8977 s^2/kg. As seen before, the equation for the slope is:

$$\text{Slope} = \frac{4\pi^2}{k} \tag{11}$$

Rearranging for k:

$$k = \frac{4\pi^2}{slope} \tag{12}$$

Therefore, solving this equation for the Line of best fit: $k = \frac{4\pi^2}{0.9652} \rightarrow k = 40.9$ N/m. The Maximum and Minimum lines are used to get the uncertainty for k. To do that, firstly it is needed to calculate the value of k for both of them, then calculate their difference (the biggest value minus the smaller value), and lastly divided it by 2. For the Maximum line: $k = \frac{4\pi^2}{1.0471} \rightarrow k = 37.7$ N/m, and for the Minimum line: $k = \frac{4\pi^2}{0.8415} \rightarrow k = 44$ N/m. The difference between the biggest and the smallest value is: $44 - 37.7 = 6.3$. Dividing by 2: $\frac{6.3}{2} = 3.15$ N/m.

Therefore, the value for the elastic constant of the dynamometer given by this experiment is:

$$k = 41 \pm 3 \text{ N/m}$$

2.8 Conclusion:

With this experiment, it could be proven algebraically and graphically, that the period squared of the mass-spring system has established a linear relationship with the mass attached to it and that this mathematical relationship depends on a constant, the elastic constant of the spring, found through graph analysis. The uncertainty for the value of the elastic constant was determined by graphical analysis of the functions of the Maximum and Minimum lines.

Lastly, the process of researching to do the experiment until reaching the final result, satisfactorily helped me to better understand and visualize this topic. For further research I would suggest making an experiment with springs where one could analyze the velocity and acceleration of a certain object, or do an experiment using pendulums, since it has a very similar equation and methodology to the one used in this experiment.

2.9 Evaluation:

<u>Percentage uncertainty of the elastic constant:</u>

$\Delta k/k \times 100\% = 3.15/40.9 \times 100\% = 7.7\%$

<u>Percentage difference</u> between the known value and the value found on the experiment:

$\frac{(known\ value - value\ found)}{known\ value} \times 100\% = \frac{(41.7-40.9)}{41.7} \times 100\% = 1.9\ \%$

<u>Range</u>

My data: 41 ± 3 N/m

Known value: 41.7 N/m

Range (my data) = 41 – 3 = 38 and 41 + 3 = 44

The minimum value possible for the elastic constant is 38 N/m, and the maximum value is 44 N/m.

Seeing that the known value is within the value of the range, that it is close to the value found, and that the expected value for b (equation 9 & 10) was 0 and the graph gave 0.0058, this experiment is completely acceptable. Even though there're uncertainties, they are still very acceptable since they are close to the known value. Therefore, it's possible to assume that the result is accurate.

2.10 Evaluation of Errors:

Table 3: Table showing the evaluation of errors

Weakness in the procedure (source of error) **How it affects results**	**Evidence of possible errors**	**Significance**	**Realistic improvement**	**Type of error**
Human reaction time error: The reaction time of a human pressing the stop button can lengthen or shortens the time of the period.	Scientific experiments have already proved that the human reaction time is circa 0.250 s. Evidence of random errors can be seen on experimental points of the	By measuring the time for ten oscillations and then dividing it by ten, the uncertainty on time measurements decreased a lot, thus having a small significance.	Instead of using a stopwatch, record a slow-motion video and use a video editing software to accurately measure the time of the oscillations.	Random

	graph being spread over and under the best-fit line.			
<u>Mass uncertainty:</u> The weighing scale has an uncertainty.	The weighing scale has an uncertainty of $\pm$ 0.001 kg	As the uncertainty is small, it can be considered insignificant.	None	Random

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